

THE UNIVERSITY OF THE WEST INDIES FIVE ISLANDS CAMPUS

Semester II

Examinations of April/May 2023

Course Code:

ECON1003

Course Title:

Mathematics for Social Sciences I

Date of Assessment:

Wednesday May 3rd, 2023

Time:

9:00 am

Duration:

Two (2) Hours

INSTRUCTIONS TO CANDIDATES:

This paper has $\underline{4}$ pages and $\underline{5}$ questions.

YOU ARE REQUIRED TO ANSWER FIVE (5) QUESTIONS.

THIS ASSESSMENT IS WORTH 60 % OF YOUR FINAL GRADE.

ASSESSMENT DETAILS FROM INSTRUCTOR(S):

- This paper contains five questions
- Each question is worth 20 marks
- Students are required to answer all questions
- Show all working clearly for each question
- The total marks is one hundred (100) marks
- Non-programmable calculators are allowed

QUESTION 1 (Total: 20 Marks)

- 1. Suppose the projected sales of UWI Student Guild t-shirts for an academic year are given by S = 100x + 4000 where x is measured in months and x = 0 corresponds to the start of the academic year.
- (a). Find the projected sales after the first semester if we consider a semester as 4 months [2 marks]
- (b). To be a profitable venture, the sales must be greater than or equal to 4500
 - (i) Express the sales equation as an inequality given the above info

[3 marks]

(ii) Find the value of x that will make the UWI Student Guild t-shirts a profitable venture

[3 marks]

- (c) Suppose that the revenue derived from the sale of q t-shirts is R(q) = 500q. If the total cost of producing q t shirts is C(q) = 100q + 6000
 - (i) Find an expression for the Profit obtained from selling q t-shirts (Recall P(q) = R(q) C(q))
 [2 marks]
 - (ii) How many t-shirts must be produced and sold to generate exactly \$20,000 in profit?[3 marks]
 - (iii)How many t-shirts must be produced and sold to break even (Total Revenue = Total Cost)?

 [3 marks]
 - (iv) Find the marginal profit when the production and sales of the t-shirts break even [2 marks]
 - (v) Will the marginal profit be constant for any quantity of shirts sold? Provide a reason to support your answer. [2 marks]

QUESTION 2 (Total 20 Marks)

2. Suppose that Caribbean United Credit Union offers the following on the latest new membership promotion for recent UWI graduates:

Offer 1

A fixed shares deposit of \$ 10,000 XCD with an annual shares deposit of \$500 XCD

Offer 2

A fixed shares deposit of \$8000 XCD with an annual shares deposit of 5% of your existing shares.

- (i) Show with supporting explanations that the first offer follows an arithmetic progression. [7 marks]
- (ii) Show with supporting explanations that the first offer follows a geometric progression[7 marks]

(iii) Compare and contrast the two offers to determine which will have yield the higher savings in shares deposit in year five of membership [4 marks]

(iv) From a strategic financial perspective, which offer is ideally the better one in the long term?

Provide supporting reasons for your response.

[2 marks]

QUESTION 3 (Total 20 Marks)

3. A function f(x) is defined as

$$\begin{cases}
5+3x & if \ x < 3 \\
2x^2-x-1 & if \ x \ge 3
\end{cases}$$

(a) Determine whether f(x) is continuous at x = 3 (Hint: you will need to consider all three conditions for continuity)

[9 marks]

(b) Evaluate the following limits

(i)
$$\lim_{x \to 4} \frac{16 - x^2}{x - 4}$$

[6 marks]

(ii)
$$\lim_{x \to \infty} \frac{10 - 7x^2 - x^3}{3x^3 - x^2 + 1}$$

[5 marks]

QUESTION 4 (Total 20 Marks)

4. **JUST 4 YOU Clothing Manufacturing Company**, the demand equation for a new denim sports jacket is given by

p = 300 - 15x where x is the quantity and p is unit price. The cost function of this item is C(x) = 1000 - 60x.

Based on the above information, compute the following for the company

(a) The revenue function R(x) (R(x) = xp, where p is unit price)

[4 marks]

(b) The profit function P(x) (P(x) = R(x) - C(x))

[4 marks]

(c) The marginal profit function

[6 marks

(d) The level of demand x which makes the marginal profit equal to zero

[3 marks]

(e) Verify that the level of demand obtained from d) above produces maximum profit for the firm.

QUESTION 5 (Total 20 marks)

5. (a) Evaluate the following integral $\int 90x^5 + 4x^3 + 2x - 5 dx$

[4 marks]

Recall the laws of Integrals

$$\int c \, dx = cx, c \text{ is a constant}$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

For any indefinite integral $\int f(x) dx = F(x) + C$, C is the constant of integration

(b) Suppose a group of Social Sciences students began creating and selling personalized hoodies to fellow students on campus. Suppose that the marginal revenue for producing x hoodies is given by $R'(x) = 30 x^2 - 4x$. Find the total revenue earned when they produce 20 hoodies if the revenue earned from producing 1 hoodie is \$50 XCD (i.e. R(1) = 50)

[8 marks]

(c) Consider the system of linear equations

$$3x + 4y = 10$$
$$2x + 3y = 7$$

(i) Write the system of linear equations below in matrix form AX = b

[3 marks]

(ii) Hence using Cramer's method, show that the solution of the equations is

$$x = 2$$
 and $y = 1$

[5 marks]

END OF QUESTION PAPER