THE UNIVERSITY OF THE WEST INDIES

FIVE ISLANDS CAMPUS

Semester II

Examinations of APRIL/MAY 2023

Course Code: **ECON1005**

Course Title: **Introduction to Statistics**

Date of Assessment: **May 5, 2023**

Time: **9:00 am**

Duration: **Two (2) Hours**

INSTRUCTIONS TO CANDIDATES:

This paper has __13_ pages and _7__ questions.

YOU ARE REQUIRED TO ANSWER ___5_____ QUESTIONS.

THIS ASSESSMENT IS WORTH _____60___ % OF YOUR FINAL GRADE.

ASSESSMENT DETAILS FROM INSTRUCTOR(S):

1. **This paper is made up of TWO Sections: A and B**
2. **Answer ALL Questions in Section A**
3. **Answer ONE Question from Section B**
4. **Show all work and draw diagrams as instructed.**
5. **Statistical Table and Formula Sheet are provided.**
6. **Silent, Cordless, non–programmable CALCULATORS are permitted.**

Section A

Answer all questions in this Section.

1. A standard normal distribution of bone density scores has a mean of 0 and standard deviation 1. Use the standard normal distribution table to find the area under the curve for the following. Draw diagrams to illustrate your answers:

   a) $-0.84 \leq z \leq 1.28$

   b) If the area to the right of z is 0.9265 find the z value

   c) The back to knee length for females sitting in an aircraft is normally distributed with a mean of 22.7 inches and standard deviation of 1.0 inches. Find the probability that a female has a back to knee length between 22.0 in. and 24.0 inches.

   (5 + 5+5 marks)

2. Assume that the population of human body temperature has a mean of 98.6◦F, as is commonly believed. Also assume that the population standard deviation is 0.62◦F (based on data from University of Maryland researchers). If a sample of size n=106 is randomly selected, find the probability of getting a mean of 98.2◦F or lower. (According to the central limit theorem sample size is greater than 30 so distribution of sample means is a normal distribution). Draw a diagram. **(5 marks)**

3. In a random sample of 25 computer scientists who subscribed to a web-based daily news update, it was found that the average salary was $46,816. Assume the population standard deviation is known to be $ 12,557 and the salaries are normally distributed. Calculate a 95 percent confidence interval for the mean salary of computer scientists. **(5 marks)**

4. A nationwide survey found that 30% of men on the block had tattoos. Suppose that this result holds true for the current population for all men on the block. Find the probability that from a random sample of 150 men on the block, 50 – 62 men have tattoos. **(5 marks)**

Hint: approximate the binomial distribution by the normal distribution and remember to use the continuity correction factor.

Section B

Answer one question in this Section.

5. It is said that happy and healthy workers are efficient and productive. A company that manufactures exercising machines wanted to know the percentage of large companies that provide on-site health club facilities. A sample of 240 such companies showed that 96 of them provide such facilities on site.

a. What is the point estimate of the percentage of all such companies that provide such facilities on site?

b. Construct a 97% confidence interval for the percentage of all such companies that provide such facilities on site. What is the margin of error for this estimate?

(2 + 8 marks)

6. The data on ***incomes*** and ***food expenditures*** of seven households is given in the Table below. *Income is the independent variable* and *food expenditure is the dependent variable.*

Income X ($'000)	**Food Expenditure** Y($'000)	XY	X^2
56	14		
84	22		
39	10		
62	15		
34	9		
50	13		
68	17		
$\sum X = 393$	$\sum Y = 100$	$\sum XY =$	$\sum X^2 =$

(a) Develop the least squares estimated regression equation based on the population regression model $Y = a + bX$

where $b = \dfrac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$ **and** $a = \overline{y} - b\overline{x}$

(3 marks)

(b) Interpret the y-intercept of the estimated regression equation from part (a) above **(1 mark)**

(c) Interpret the slope of the estimated regression equation from part (a) above. **(1 mark)**

(d) Predict the Food Expenditure when Income is $34,000 **(2 marks)**

(e) Predict the Income when Food Expenditure is $10,000 **(2 marks)**

(f) If r , the correlation coefficient is 0.994, interpret what this means.

(1 mark)

7. a) List three properties of the binomial distribution **(3 marks)**

b) In a large corporation, 65% of the employees are male. A random sample of five employees is selected. Use your knowledge of the binomial probability distribution to answer the following questions.

i) Use the formula to find the probability that the sample contains exactly three male employees? **(3 marks)**

ii) Calculate the mean (expected value) number of males in the sample? **(1 mark)**

iii) Calculate the standard deviation of the number of males in the sample. **(2 marks)**

c) Ms. Jarvis sells mangos at the gate at Ottos Comprehensive High School. On any given day, the probability of Ms. Jarvis selling 0 mangos is 0.1, 1 mango is 0.8 and 2 mangos is 0.1. Construct the probability distribution table for Ms. Jarvis's mango sales. **(1 mark)**

END OF QUESTION PAPER

Formula Sheet

Discrete Random Variables

- Mean of a discrete random variable X

$$\mu = E(X) = \sum xP(x)$$

- Standard deviation of a discrete random variable X

$$\sigma = \sqrt{\Sigma x^2 P(x) - \mu^2}$$

Alternatively, we can use the formula:

$$\sigma = \sqrt{\sum (x - \mu)^2 P(x)}$$

- Probability distribution function (Binomial variable)

$$P(X) = {}^{n}C_x \, p^x q^{n-x}$$

- Mean or Expected value of a binomial random variable

$$\mu = E(X) = np$$

- Standard deviation of a binomial random variable

$$\sigma = \sqrt{npq}$$

Continuous Random Variables - Standard Normal distribution

- Finding the corresponding Z value

$$Z = \frac{X - \mu}{\sigma}$$

- Finding the value of X given μ, σ and Z :

$$X = \mu + Z\sigma$$

Estimation of the Mean and Proportion

- Confidence interval for μ

$$\bar{X} \pm Z\frac{\sigma}{\sqrt{n}}$$

$$\bar{X} \pm t\frac{s}{\sqrt{n}}$$

- Confidence interval for p (large samples)

$$\hat{p} \pm Z\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

- Determining the sample size for estimating μ

$$n = \frac{Z^2\,\sigma^2}{ME^2}$$

where ME is the margin of error

- Determining the sample size for estimating p

$$n = \frac{Z^2 \hat{p}\hat{q}}{ME^2}$$

where ME is the margin of error

Hypotheses test about a mean

- Test statistics

$$Z_{test} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$t_{test} = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$$

Degrees of freedom for a t-test $df = n - 1$

Chi-square test of independence

- Degrees of freedom for a test of independence

$$df = (r-1)(c-1)$$

- Expected values

$$E = \frac{\text{Row total} \times \text{Column total}}{n}$$

- Test statistics

$$\chi^2_{test} = \sum \frac{(O-E)^2}{E}$$

Correlation and simple linear regression

- Linear correlation coefficient

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}\, SS_{yy}}}$$

where

$$SS_{xy} = \sum XY - \frac{(\sum X)(\sum Y)}{n}$$

$$SS_{xx} = \sum X^2 - \frac{(\sum X)^2}{n}$$

$$SS_{yy} = \sum Y^2 - \frac{(\sum Y)^2}{n}$$

- Least squares estimates of the intercept and slope of a simple linear regression

$$\text{Intercept: } a = \bar{Y} - b\bar{X}$$

$$\text{Slope: } b = \frac{SS_{xy}}{SS_{xx}}$$

Table IV Standard Normal Distribution Table

The entries in the table on this page give the cumulative area under the standard normal curve to the left of z with the values of z equal to 0 or negative.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

(continued on next page)

Table IV Standard Normal Distribution Table (continued from previous page)

The entries in the table on this page give the cumulative area under the standard normal curve to the left of z with the values of z equal to 0 or positive.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

This is Table IV of Appendix C.