



**THE UNIVERSITY OF THE WEST INDIES
FIVE ISLANDS CAMPUS**

Semester II

Examinations of April/May 2022

Course Code: MATH0100
Course Title: Pre-Calculus
Date of Assessment: 3rd May, 2022
Time: 1:00 PM
Duration: Two (2) Hours

INSTRUCTIONS TO CANDIDATES:

This paper has 3 pages and 5 questions.

YOU ARE REQUIRED TO ANSWER ALL (5) QUESTIONS.

THIS ASSESSMENT IS WORTH 50 % OF YOUR FINAL GRADE.

Examination Weight: 50%

Marked Out of: 100

ASSIGNMENT DETAILS FROM INSTRUCTOR:

- 1. This examination consists of FIVE (5) questions.**
- 2. Answer ALL questions.**
- 3. Write your answers on binder sheets.**
- 4. Please ensure that your name and ID number is written on each page.**
- 5. ALL calculations/working MUST be clearly shown.**
- 6. Silent, Cordless, Non-Programmable CALCULATORS are permitted.**

1. (a) Given position vectors $\overrightarrow{OF} = \begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix}$ and $\overrightarrow{OG} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$. Find:

(i) $|\overrightarrow{OF}|$, the magnitude of \overrightarrow{OF} . [2 marks]

(ii) \overrightarrow{FG} . [2 marks]

(iii) Find the (vector) equation of the line through the point F that is parallel to FG. [6 marks]

Note: Vector equation of a line is $r = r_0 + t.v$ where t is the parameter and v is the vector which is parallel to the line.

(b) Given position vectors $\overrightarrow{OL} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$ and $\overrightarrow{OM} = \begin{pmatrix} 4 \\ -2 \\ 7 \end{pmatrix}$. Find:

(i) \overrightarrow{LM} . [2 marks]

(ii) Write \overrightarrow{LM} in the unit vector form $xi + yj + zk$ [2 marks]

(iii) Find the (vector) equation of the plane which passes through the point L (5, 1, 3) and which is perpendicular to the vector \overrightarrow{LM} . [6 marks]

Note: Vector equation of a plane is $r \cdot n = a \cdot n$ where (n) is the perpendicular vector and (a) is the position vector of a point on the plane.

Total [20 marks]

2. (a) Evaluate the following limits:

(i) $\lim_{x \rightarrow 2} x^3$ [2 marks]

(ii) $\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x^2 + 3x - 4}$ [4 marks]

(b) Evaluate the following limits:

(i) $\lim_{x \rightarrow \infty} \frac{3}{x+2}$ [2 marks]

(ii) $\lim_{x \rightarrow \infty} \frac{4 - 5x^3}{7x + 8x^3}$ [4 marks]

(c) A function $f(x)$ is defined by:

$$f(x) = \begin{cases} 2 + 3x & \text{if } x < 2 \\ 4x & \text{if } x \geq 2 \end{cases}$$

Evaluate the following:

(i) $\lim_{x \rightarrow 2^-} f(x)$ (ii) $\lim_{x \rightarrow 2^+} f(x)$ (iii) $\lim_{x \rightarrow 0} f(x)$ [6 marks]

(ii) Is $f(x)$ continuous at $x = 2$? Give a reason. [2 marks]

Total [20 marks]

3. (a) Given that $y = 3x^4 - 2x^{-1} - x + 5$, find:

(i) $\frac{dy}{dx}$ [3 marks]

(ii) $\frac{d^2y}{dx^2}$ [2 marks]

(b) (i) Find the gradient of the curve $f(x) = 4x^5 - 5x^3 + 6x^2 - x + 8$ at $x = 1$. [3 marks]

(ii) State whether the function $f(x)$ is increasing, decreasing or stationary. Justify your answer. [2 marks]

(c) Given that $y = (3x^2 - 5x)(x^3 + 2x - 4)$, find $\frac{dy}{dx}$. [5 marks]

Hint: For Product Rule, $\frac{d}{dx}(uv) = v.u' + u.v'$

(d) Given that $y = \frac{3x^2 - 5x}{x^3 + 2x - 4}$, find $\frac{dy}{dx}$. [5 marks]

Hint: For Quotient Rule, $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.u' - u.v'}{v^2}$

Total [20 marks]

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4. (a) If $y = (1 - 4x^3)^5$, find $\frac{dy}{dx}$. [5 marks]

(b) (i) Find the gradient of the tangent to the curve $f(x) = 3x^2 - x - 4x^{-1}$ at $x = 1$. [3 marks]

(iii) Hence, find the equation of the tangent to the curve in the form $y = mx + c$ or $y - y_1 = m(x - x_1)$ [3 marks]

where $m =$ gradient of the tangent, $x_1 = 1$ and $y_1 = f(1)$.

(c) Find the stationary point(s) of the function $f(x) = x^3 + 3x^2 - 9x + 4$, and determine their nature (or classification). [9 marks]

Total [20 marks]

5. (a) Find $\int (5x^4 - 4x) dx$. [4 marks]

(b) Evaluate $\int_1^4 (2x - 3) dx$. [5 marks]

(c) Find the area enclosed by the curve $y = 3x^2$, the lines $x = 1$ and $x = 3$ and the x -axis. [6 marks]

Hint: Area enclosed = $\int_a^b (y) dx$

(d) Find the equation of the curve that passes through the point (1, 5) and for which $y = \int (6x^2) dx$. [5 marks]

Total [20 marks]

END OF QUESTION PAPER