



**THE UNIVERSITY OF THE WEST INDIES  
FIVE ISLANDS CAMPUS**

Semester II

**Examinations of April/May 2023**

**Course Code: MATH0100**

**Course Title: Pre-Calculus**

**Date of Assessment: 27<sup>th</sup> April 2023**

**Time: 4:00 pm**

**Duration: Two (2) Hours**

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**INSTRUCTIONS TO CANDIDATES:**

This paper has 4 pages and 5 questions.

**YOU ARE REQUIRED TO ANSWER ALL (5) QUESTIONS.**

**THIS ASSESSMENT IS WORTH 50 % OF YOUR FINAL GRADE.**

**Examination Weight: 50%**

**Marked Out of: 100**

**ASSESSMENT DETAILS FROM INSTRUCTOR(S):**

- 1. This examination consists of FIVE (5) questions.**
- 2. Answer ALL questions.**
- 3. Please ensure that your ID number is written on each page.**
- 4. ALL calculations/working MUST be clearly shown.**
- 5. Silent, Cordless, Non-Programmable CALCULATORS are permitted.**

1. (a) Given position vectors  $\overrightarrow{OL} = \begin{pmatrix} 8 \\ -1 \\ 5 \end{pmatrix}$  and  $\overrightarrow{OM} = \begin{pmatrix} -4 \\ 0 \\ 1 \end{pmatrix}$ . Find:

- (i)  $|\overrightarrow{OL}|$ , the magnitude of  $\overrightarrow{OL}$ . [2 marks]  
 (ii)  $\overrightarrow{LM}$ . [2 marks]  
 (iii) Find the (vector) equation of the line through the point L that is parallel to LM. [6 marks]

**Note:** Vector equation of a line is  $r = r_0 + t.v$  where  $t$  is the parameter and  $v$  is the vector which is parallel to the line.

(b) Given position vectors  $\overrightarrow{OR} = \begin{pmatrix} 6 \\ 2 \\ -3 \end{pmatrix}$  and  $\overrightarrow{OT} = \begin{pmatrix} 5 \\ 3 \\ 7 \end{pmatrix}$ . Find:

- (i)  $\overrightarrow{RT}$ . [2 marks]  
 (ii) Write  $\overrightarrow{RT}$  in the unit vector form  $xi + yj + zk$  [2 marks]  
 (iii) Find the (vector) equation of the plane which passes through the point R (6, 2, -3) and which is perpendicular to the vector  $\overrightarrow{RT}$ . [6 marks]

**Note:** Vector equation of a plane is  $r \cdot n = a \cdot n$  where  $(n)$  is the perpendicular vector and  $(a)$  is the position vector of a point on the plane.

2. (a) Evaluate the following limits:

- (i)  $\lim_{x \rightarrow -2} \frac{1}{x^3}$  [2 marks]  
 (ii)  $\lim_{x \rightarrow 1} \frac{x^2 - 7x + 12}{x^2 - x - 6}$  [4 marks]

(b) Evaluate the following limits:

- (i)  $\lim_{x \rightarrow \infty} \frac{9}{8 + x}$  [2 marks]  
 (ii)  $\lim_{x \rightarrow \infty} \frac{3 + 4x^2}{x - 5x^2}$  [4 marks]



- (c) A function  $f(x)$  is defined by:

$$f(x) = \begin{cases} x^2 - 2 & \text{if } x \geq 4 \\ 3x & \text{if } x < 4 \end{cases}$$

Evaluate the following:

(i)  $\lim_{x \rightarrow 4^-} f(x)$       (ii)  $\lim_{x \rightarrow 4^+} f(x)$       (iii)  $\lim_{x \rightarrow 0} f(x)$       **[6 marks]**

- (ii) Is  $f(x)$  continuous at  $x = 4$ ? Give a reason.      **[2 marks]**

3. (a) Given that  $y = 5x^3 - 4x^{-2} + x - 6$ , find:

(i)  $\frac{dy}{dx}$       **[3 marks]**

(ii)  $\frac{d^2y}{dx^2}$       **[2 marks]**

- (b) (i) Find the gradient of the curve  $f(x) = 8x^3 + 7x^2 - 10x + 6$  at  $x = -1$ .      **[3 marks]**

- (ii) State whether the function  $f(x)$  is increasing, decreasing or stationary. Justify your answer.      **[2 marks]**

- (c) Given that  $y = (7x^2 - 4x)(x^3 - 5x + 3)$ , find  $\frac{dy}{dx}$ .      **[5 marks]**

**Hint: For Product Rule,  $\frac{d}{dx}(uv) = v.u' + u.v'$**

- (d) Given that  $y = \frac{7x^2 - 4x}{x^3 - 5x + 3}$ , find  $\frac{dy}{dx}$ .      **[5 marks]**

**Hint: For Quotient Rule,  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.u' - u.v'}{v^2}$**

4. (a) If  $y = (5 - 7x^3)^4$ , find  $\frac{dy}{dx}$ .      **[5 marks]**

- (b) (i) Find the gradient of the tangent to the curve  $f(x) = 2x^3 - x^2 + 5x + 1$  at  $x = 2$ .      **[3 marks]**

- (ii) Hence, find the equation of the **normal** to the curve in the form  $y = mx + c$  **or**  $y - y_1 = m(x - x_1)$  **[3 marks]**  
where  $m =$  gradient of the normal,  $x_1 = 2$  and  $y_1 = f(2)$ .

- (c) Find the stationary point(s) of the function  $f(x) = x^3 - 6x^2 - 15x + 1$ , and determine their nature (or classification). **[9 marks]**

5. (a) Find  $\int (8x^3 - 12x^2 + 3x) dx$ . **[4 marks]**

- (b) Evaluate  $\int_1^4 (6x^2 - 5) dx$ . **[5 marks]**

- (c) Find the area enclosed by the curve  $y = 8x - 6$ , the lines  $x=1$  and  $x=5$  and the  $x$ -axis. **[6 marks]**

Hint: Area enclosed = $\int_a^b (y) dx$
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- (d) Find the equation of the curve that passes through the point  $(1, 7)$  and for which  $y = \int (12x^2 - 3) dx$ . **[5 marks]**

**END OF QUESTION PAPER**