



**THE UNIVERSITY OF THE WEST INDIES
FIVE ISLANDS CAMPUS**

Semester II

Examinations of April/May 2022

Course Code: MATH0110
Course Title: Calculus and Analytical Geometry
Date of Assessment: 5th May, 2022
Time: 1:00 PM
Duration: Two (2) hours

INSTRUCTIONS TO CANDIDATES:

This paper has 3 pages and 5 questions.

YOU ARE REQUIRED TO ANSWER ALL (5) QUESTIONS.

THIS ASSESSMENT IS WORTH 50 % OF YOUR FINAL GRADE.

Examination Weight: 50% Marked Out of: 100

ASSIGNMENT DETAILS FROM INSTRUCTOR:

- 1. This examination consists of FIVE (5) questions.**
- 2. Answer ALL questions.**
- 3. Write your answers on binder sheets.**
- 4. Please ensure that your name and ID number is written on each page.**

1. (a) Find the 2nd degree Taylor polynomial for $f(x) = \ln x$ centered at $c = 3$ and use it to approximate $\ln(3.3)$.

[10 marks]

Note: Taylor's Theorem is:

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \dots + \frac{f^{(r)}(c)(x-c)^r}{r!} + \dots$$

- (b) Find the 3rd degree Maclaurin Series polynomial for $f(x) = e^x$ and use it to approximate $e^{1.0}$.

[10 marks]

Hint: Maclaurin Series is a special type of Taylor series which is always centered at $c = 0$

Note:
$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots + \frac{f^{(r)}(0)x^r}{r!} + \dots$$

Total [20 marks]

2. (a) Show that the equation $x^3 - 3x^2 - 1 = 0$ has a root, α , between 3 and 4. Justify your answer.

[6 marks]

- (b) Show that the function $f(x) = x^3 - 3x^2 - 1$ is increasing at $x=3$.

[2 marks]

- (c) **Taking the number 3 as the first approximation for α** , use the Newton-Raphson method to find a better approximation for α , to 3 **decimal places**.

[12 marks]

Total [20 marks]

Note: The Newton-Raphson formula is:If x_r is an approximation to a root, α , of the equation $f(x) = 0$, then:

$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)} \text{ gives a better approximation to } \alpha$$

3. (a) Find the number of ways of selecting (choosing) 4 persons from a list of 7 persons to sit on a committee.

[2 marks]

- (b) How many different five (5)-digit codes can be made from the digits 3, 4, 5, 6, 7, 8, 9 if each digit can only be used once?

[2 marks]

- (c) In how many ways can the letters in the word NUMBER be arranged?

[2 marks]

- (d) In a pack of cards, find the probability of obtaining:
a Nine **OR** a Heart i.e. $P(9 \cup \text{heart})$

[3 marks]

3. (f) A survey of a sample of business students resulted in the following information regarding the genders of the individuals and their selected major.

| Gender | Selected Major | | | Total |
|--------------|----------------|-----------|--------|-------|
| | Management | Marketing | Others | |
| Male | 40 | 10 | 30 | 80 |
| Female | 30 | 20 | 70 | 120 |
| Total | 70 | 30 | 100 | 200 |

- i. What is the probability of selecting a female? **[2 marks]**
- ii. What is the probability of selecting a male who is also majoring in Marketing? **[2 marks]**
- iii. What is the probability of selecting an individual who is majoring in Marketing, given that the person is female? **[2 marks]**
- iv. What is the probability of selecting a male OR a management major? **[3 marks]**

Total [20 marks]

4. (a) Find the Inverse M^{-1} , of the Matrix $M = \begin{pmatrix} 1 & -3 & 0 \\ 2 & 0 & 1 \\ 4 & 1 & 3 \end{pmatrix}$. **[10 Marks]**

- (b) Solve the Simultaneous Equations using Row Echelon Form (REF)

$$x + y + z = 5$$

$$2x + 2y - z = 7$$

$$x - y + 2z = 2$$

[8 Marks]

- (c) Are the systems of equations in 4 (b) above, Consistent or Inconsistent? Justify your answer. **[2 Marks]**

Total [20 marks]

5. (a) Solve the (First Order) differential equation $(x^4 - 7) \frac{dy}{dx} = 4x^3$. **[9 Marks]**

- (b) Solve the differential equation $\frac{d^2y}{dx^2} + 20 \frac{dy}{dx} + 64y = 0$