



**THE UNIVERSITY OF THE WEST INDIES
FIVE ISLANDS CAMPUS**

Semester II

Examinations of April/May 2023

Course Code: MATH0110
Course Title: Calculus and Analytical Geometry
Date of Assessment: 5th May 2023
Time: 9:00 am
Duration: Two (2) Hours

INSTRUCTIONS TO CANDIDATES:

This paper has 4 pages and 5 questions.

YOU ARE REQUIRED TO ANSWER ALL (5) QUESTIONS.

THIS ASSESSMENT IS WORTH 50 % OF YOUR FINAL GRADE.

Examination Weight: 50%

Marked Out of: 100

ASSESSMENT DETAILS FROM INSTRUCTOR(S):

- 1. This examination consists of FIVE (5) questions.**
- 2. Answer ALL questions.**
- 3. Please ensure that your ID number is written on each page.**
- 4. ALL calculations/working **MUST** be clearly shown.**
- 5. Silent, Cordless, Non-Programmable CALCULATORS are permitted.**

1. (a) Find the 3rd degree Taylor polynomial for $f(x) = \ln x$ centered at $c = 5$ and use it to approximate $\ln(5.6)$ correct to 4 decimal places.

[10 marks]

Note: Taylor's Theorem is:

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \frac{f'''(c)(x-c)^3}{3!} + \dots + \frac{f^r(c)(x-c)^r}{r!} + \dots$$

- (b) Find the 4th degree Maclaurin Series polynomial for $f(x) = e^x$ and use it to approximate $e^{1.5}$ correct to 4 decimal places.

[10 marks]

Hint: Maclaurin Series is a special type of Taylor series which is always centered at $c = 0$

Note:
$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{(4)}(0)x^4}{4!} + \dots + \frac{f^r(0)x^r}{r!} + \dots$$

2. (a) Show that the equation $2x^3 - 6x^2 - 2 = 0$ has a root, α , between 3 and 4. Justify your answer.

[6 marks]

- (b) Show that the function $f(x) = 2x^3 - 6x^2 - 2$ is increasing at $x=4$.

[2 marks]

- (c) Taking the number 4 as the first approximation for a root of the equation α , use the Newton-Raphson method to find a better approximation for α , correct to 3 decimal places.

[12 marks]

Note: The Newton-Raphson formula is:If x_r is an approximation to a root, α , of the equation $f(x) = 0$, then:

$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)} \text{ gives a better approximation to } \alpha$$

3. (a) Find the number of ways of selecting (choosing) 6 persons from a list of 9 persons to sit on a committee.

[2 marks]

- (b) How many different five (5)-digit codes can be made from the digits 2, 3, 4, 5, 6, 7, 8, 9 if each digit can only be used once?

[2 marks]

- (c) In how many ways can the letters in the word CALCULUS be arranged? **[2 marks]**
- (d) In a pack of cards, find the probability of obtaining:
an Seven **OR** a Diamond i.e. $P(7 \cup \text{Diamond})$. **[3 marks]**
- (e) If X and Y are **mutually exclusive events** with $P(X) = 0.30$ and $P(Y) = 0.40$,
find $P(X|Y)$. **[2 marks]**
- (f) A survey of a sample of business students resulted in the following
information regarding the genders of the individuals and their selected
major.

Gender	Selected Major			Total
	Artificial Intelligence	Computer Science	Others	
Male	80	20	60	160
Female	60	40	140	240
Total	140	60	200	400

- i. What is the probability of selecting a male? **[2 marks]**
- ii. What is the probability of selecting a female who is also majoring in
Computer Science? **[2 marks]**
- iii. What is the probability of selecting an individual who is majoring in
Computer Science, given that the person is male? **[2 marks]**
- iv. What is the probability of selecting a female **OR** an Artificial
Intelligence major? **[3 marks]**

4. (a) Find the Inverse M^{-1} , of the Matrix $M = \begin{pmatrix} 1 & -1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & 4 \end{pmatrix}$. **[10 Marks]**

- (b) Solve the Simultaneous Equations using Row Echelon Form (REF)

$$x + y + z = 2$$

$$2x + y - 3z = 1$$

$$2x - y - 5z = 3$$

[8 Marks]

- (c) Are the systems of equations in 4 (b) above, Consistent or Inconsistent? Justify your answer. [2 Marks]

5. (a) Solve the (First Order) differential equation $(x^6 + 4)\frac{dy}{dx} = 6x^5$. [9 Marks]

- (b) Solve the differential equation $\frac{d^2y}{dx^2} - 17\frac{dy}{dx} + 16y = 0$,

[in the form $y = Ae^{\alpha x} + Be^{\beta x}$], given that when $x = 0, y = 0$ and $\frac{dy}{dx} = 0.6$.

[11 Marks]

END OF QUESTION PAPER