THE UNIVERSITY OF THE WEST INDIES
FIVE ISLANDS CAMPUS

Semester II

Examinations of April/May 2023

Course Code:	**ECON2016**
Course Title:	**Mathematics for Social Sciences III**
Date of Assessment:	**Monday April 24th, 2023**
Time:	**9:00 am**
Duration:	**Two (2) Hours**

INSTRUCTIONS TO CANDIDATES:

This paper has 3 pages and 4 questions.

YOU ARE REQUIRED TO ANSWER FOUR QUESTIONS.

THIS ASSESSMENT IS WORTH 60 % OF YOUR FINAL GRADE.

ASSESSMENT DETAILS FROM INSTRUCTOR(S):

- **This paper contains four questions**
- **Each question is worth 15 marks**
- **Students are required to answer all questions**
- **Show all working clearly for each question**
- **The total marks is sixty (60) marks**
- **Non-programmable calculators are allowed**

QUESTION 1 (Total: 15 marks)

1. (a) Describe the following ϵ-neighbourhoods and represent them graphically

(i) $N_\epsilon(5)$ **[3 marks]**

(ii) $N_\epsilon[(5,3)]$ **[3 marks]**

(iii)$N_\epsilon[(5,3,1)]$ **[4 marks]**

(b) Prove that the intersection of two closed sets is closed **[5 marks]**

QUESTION 2 (Total 15 marks)

2. (a)Define the convex combination of two points x and x' and explain what this represents graphically. **[3 marks]**

(b) Find the convex combination of the pair of points in $\mathbb{R}^4$ $(2,1,0,-2)$ and $(-3,2,1,5)$ when $\lambda = 2/3$ **[3 marks]**

(c)Suppose that the revenue function and cost function for the production of a new blue-tooth ear plug is given by $R(x) = 20 - x^2$ and $C(x) = x^2$.

(i) Prove that the function $R(x)$ is concave **[5 marks]**

(ii) Show that the function C(x) is convex **[4 marks]**

QUESTION 3 (Total 15 marks)

3. Suppose that the revenue function for the sale of burgers at Red Apple Café is given by

$$R\,(x) = (4x - \frac{1}{60}x^2)$$

(a) Show that this function is strictly convex **[5 marks]**

(b) Suppose that a manufacturing model of three types of canned virgin margaritas at Red Apple Cafe is represented by the following function $f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + x_3^2 + x_1x_2 - 2x_3 - 7x_1 + 12$. Determine the maxima and minima (if any) of the function. **[10 marks]**

QUESTION 4 (Total 15 marks)

4.(a) Suppose that an economic growth model for agriculture production are patterned by various differentials equations. For each of the following differential models find the solution of the equation. The variable y represents the profit obtained when quantity x of harvested crops such as sweet potatoes or yams are sold.

(a) $\frac{dy}{dx} = \frac{y^2+xy^2}{x^2y-x^2}$ **[4 marks]**

(b) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 2e^x$ **[5 marks]**

(b) Find the marginal-product functions for the Cobb-Douglas production function **[6 marks]**

$$y = 100x_1^{1/2}x_2^{1/3}x_3^{1/4}$$

END OF QUESTION PAPER