



**THE UNIVERSITY OF THE WEST INDIES
FIVE ISLANDS CAMPUS**

Semester II

Examinations of APRIL/MAY 2023

Course Code: ECON2026
Course Title: Statistical Methods II
Date of Assessment: Friday May 5, 2023
Time: 9:00 am
Duration: Two (2) Hours

INSTRUCTIONS TO CANDIDATES:

This paper has 3 pages and 3 questions.

STUDENTS ARE REQUIRED TO ANSWER ALL QUESTIONS.

THIS ASSESSMENT IS WORTH 60 % OF YOUR FINAL GRADE.

ASSESSMENT DETAILS FROM INSTRUCTOR(S):

All working must be clearly shown. Electronic Non-Programmable Calculators may be used in the examination. Students may submit their answers in black or blue ink (No work is to be submitted in pencil). Begin the answer to each question on a new page.

STUDENTS ARE REQUIRED TO ANSWER ALL QUESTIONS

1. Let $\hat{\sigma}^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n}$ be one estimator of the infinite population variance, σ^2 .

The mean of the infinite population is μ .

(a) Show that the estimator is biased. **(10 marks)**

(b) Show that the estimator is asymptotically unbiased. **(5 marks)**

(c) Show that the estimator is consistent. **(10 marks)**

2. Consider the following model

$$y_t = \beta_1 + \beta_2 x_t + u_t$$

Where t is time index, y_t and x_t are the variables of interest and $u_t \sim IN(0, \sigma^2)$

a. Obtain the maximum likelihood estimators of the parameters of the model

(15 marks)

b. Compare and contrast the above method of estimation with the ordinary least squares method

(10 marks)

3. The estimation of a Cobb-Douglas production function for **TWENTY** (20) firms of a given industry yields:

$$\hat{y}_i = -16.907 + 0.332k_i + 2.77 l_i$$

$$R^2 = 0.915 \text{ RSS} = 0.461$$

$$\text{Var } \hat{\beta} = \begin{pmatrix} 4.939971 & & & \\ 0.216639 & 0.031875 & & \\ -0.857446 & -0.057229 & 0.166070 & \end{pmatrix}$$

\hat{y}_i = fitted value of natural logarithm of output (expressed in 1000 tons)

l_i = natural logarithm of labour (labour in hours)

k_i = natural logarithm of capital (Capital expressed in machine hours)

$\text{Var } \hat{\beta}$ = variance-covariance matrix of estimates

i - firm index

The set of critical values to choose from is:

$$\text{One-sided } t_{\alpha, \nu} : t_{0.05, 20} = 1.725, t_{0.05, 17} = 1.740, t_{0.05, 18} = 1.734$$

$$\text{Double-sided } t_{\alpha, \nu} : t_{0.025, 20} = 2.086, t_{0.025, 17} = 2.110, t_{0.025, 18} = 2.101$$

$$\text{At the 5\% level of significance, } F_{20}^2 = 3.49; F_{18}^3 = 3.16; F_{17}^2 = 3.59$$

- Provide and explain three reasons for the existence of the error term. Moreover, explain the difference between " u " and " \hat{u} ". **(7 marks)**
- b) Using a 5% level of significance, test whether each variable individually affects output and carefully interpret the results. Note: the student must state the null and alternative hypotheses, the test statistic of interest, the critical value(s) and the decision. **(6 marks)**
- Using a 5% level of significance, test whether all variables taken together significantly affect output. The note in (b) also applies here. **(6 marks)**
- Using a 5% level of significance, test whether production is characterized by constant returns to scale. Explain your answer. Note that $\text{Cov}(\hat{\beta}_1, \hat{\beta}_2) = -0.135041$. **(6 marks)**

END OF QUESTION PAPER