



**THE UNIVERSITY OF THE WEST INDIES  
FIVE ISLANDS CAMPUS**

Semester II

**Examinations of APRIL/MAY 2023**

**Course Code:** FINA2002  
**Course Title:** Quantitative Methods for Banking and Finance  
**Date of Assessment:** April 25, 2023  
**Time:** 9:00 am  
**Duration:** Two (2) Hours

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**INSTRUCTIONS TO CANDIDATES:**

This paper has 12 pages and 6 questions.

**YOU ARE REQUIRED TO ANSWER 3 QUESTIONS.**

**THIS ASSESSMENT IS WORTH 60 % OF YOUR FINAL GRADE.**

**ASSESSMENT DETAILS FROM INSTRUCTOR(S):**

**This paper has 3 Sections, A, B & C. Answer 1 question from each section. A silent non-programmable electronic calculator is allowed. Statistical tables will be provided.**

## SECTION A

## Question 1

- (i) Sandra is considering purchasing a financial asset that will pay \$3 000 per year for 8 years, with the first payment 1 year from now. The required rate of return is 12% per year. How much should she pay for this asset? [4]
- (ii) The People's Bank offers to pay you 9% per annum compounded quarterly. If you invest \$40 000, what is the value of the investment in 4 years if interests are reinvested at 9%? What would have been the value of the investment if the interest was compounded continuously? [4]
- (iii) For your retirement today, your company is offering you an immediate lump sum of \$1.2 million or an annual annuity of 18 payments of \$175 000 starting today. Your bank is offering you an interest rate of 7% per year compounded yearly. Which option has the greater present value? Justify your answer. [4]
- (iv) A pension fund anticipates that benefits of \$1 500 000 per year must be paid to retirees. Retirements will commence 9 years from now at  $t = 9$ . The benefit payments will continue until  $t = 25$ . What is the present value of the pension liability if the annual discount rate is 5% compounded annually? [4]
- (v) The manager of a pension fund knows that the fund must make a payment of 8 million dollars 12 years from now. She wants to make an investment today so that it will grow to the required amount. The current interest rate is 8% per year compounded every 2 months. How much should she invest? [4]



## Question 2

(a) Give the meaning of each of the following terms:

- (i) Effective Annual Interest Rate (EAR) [1]
- (ii) Stated Annual Interest Rate [1]
- (iii) Annuity [1]
- (iv) Perpetuity [1]

(b) A company's assets grew from \$1 537 205 in 2015 to \$3 213 841 in 2021.

Calculate the average annual growth rate of the company's assets. [3]

(c) (i) Define Net Present Value (NPV) and state the NPV rule. [3]

(ii) Define Internal Rate of Return (IRR) and state the IRR rule. [3]

(d) (i) You are looking at investing \$1 800 000. This investment will generate \$400 000 at the end of year 1, \$450 000 at the end of year 2, and \$600 000 at the end of year 3. Using 7.5% as the discount rate, calculate the NPV.

Should you accept this investment? Why or why not? [4]

(ii) Determine the IRR for an initial outlay of \$3.5 million, and a cash flow of

\$275 000 per year into perpetuity. [3]

## SECTION B

## Question 3

A Manufacturing company must decide whether to make a part or to purchase it. The following payoff table shows the predicted profits in thousands of dollars.

Decisions	State of Nature		
	Low	Medium	High
	Demand	Demand	Demand
	L	M	H
Make, m	-20	50	90
Buy, b	15	55	65

The state-of-nature probabilities are  $P(L) = 0.5$ ,  $P(M) = 0.4$ ,  $P(H) = 0.1$ .

- Use a decision tree to make a recommendation. [5]
- Find the EVPI and use it to determine if the company should obtain a better estimate of demand. [3]
- A test market study of the potential demand for the part is expected to report either favourable ( $F$ ) or unfavourable ( $U$ ) condition. The conditional probabilities are as follows:

$$\begin{array}{ll}
 P(F|L) = 0.20 & P(U|L) = 0.80 \\
 P(F|M) = 0.30 & P(U|M) = 0.70 \\
 P(F|H) = 0.10 & P(U|H) = 0.90
 \end{array}$$

- What is the probability that the market research report will be favourable? [3]
- What is the company's optimal decision strategy? [5]
- What is the expected value of the market research information? [2]
- What is the efficiency of the information? [2]



## Question 4

- (a) A money manager wants to obtain a 95% confidence interval for percentage cash inflows and outflows over the next 8 months for his clients into a fund. He begins by calling a random sample of 10 clients, asking them about their planned additions to and withdrawals from the fund in the next 8 months. Inflows are considered positive, while outflows are considered negative. As a result, the money manager calculates an average cash flow of 7.0 % with a standard deviation for the sample of 11%. It may be assumed that the population is approximately normal. Calculate a 95% confidence interval and give an interpretation. [4]
- (b) A growth fund that has been in existence for 18 months was expected to earn a 2.30 percent mean monthly return during that time period. However, during that period, it has received a mean monthly return of 3.00 percent, with a sample standard deviation of 2.80 percent. Assuming returns are normally distributed, are the results consistent with a population mean monthly return of 2.30 percent? Use the 1% level of significance. [8]
- (c) The table below gives the mean monthly Return and Standard Deviation for 2 time periods.

Time Period	Number of months	Mean Monthly Return (%)	Standard Deviation
2015 through 2016	24	1.589	3.987
2017 through 2018	24	0.732	3.725

Assume equal population variances for returns in the two time periods. Are the mean monthly returns in the 2 time periods significantly different from each other? Test at the 1% level of significance. [8]

## SECTION C

## Question 5

The table below shows the predicted average percentage change of an economic variable and the actual average percentage change over a period of 7 months.

Month	1	2	3	4	5	6	7
(x) Predicted (%)	0.3	0.6	0.9	1.0	1.2	1.3	1.4
(y) Actual (%)	0.1	0.2	0.2	1.1	1.4	1.6	1.6

- (a) Calculate Pearson's Product-Moment Correlation Coefficient,  $r$ , and give an interpretation. [6]
- (b) Calculate the coefficient of determination and give an interpretation [2]
- (c) Determine the regression equation  $\hat{y} = b_0 + b_1x$  [5]
- (d) Interpret the values of the regression coefficients  $b_0$  and  $b_1$  from (c) above [4]
- (e) Use the regression equation to predict the actual average percentage change when the predicted percentage change is 0.8 %. [3]

## Question 6

The 52-week percentage change in a random sample of stock indexes for 3 countries A, B and C is given in the table below for a 6-year period.

Year	1	2	3	4	5	6
Country A	4.5	0.7	6.1	1.3	2.6	4.2
Country B	0.9	2.4	1.7	3.9	5.1	1.5
Country C	1.2	3.3	1.5	0.4	4.1	2.4

Assuming that the data constitute random samples from 3 normal populations with equal variances, test at the 5% level of significance whether the differences among the three sample means can be attributed to chance.

- (i) State the Null and Alternative Hypotheses. [2]
- (ii) State the Distribution. [2]
- (iii) State the Level of the test. [2]



- (iv) State the Rejection Criterion (Decision Rule) [3]
- (v) Calculate the Test Statistic. [8]
- (vi) State the conclusion. [3]

You may use the following ANOVA table to assist you.

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F
Treatments	$k - 1$	$SS(Tr)$	$MS(Tr)$	$\frac{MS(Tr)}{MSE}$
Error	$k(n - 1)$	$SSE$	$MSE$	
Total	$kn - 1$	$SST$		

### END OF QUESTION PAPER

### Formulae

- Future Value:  $FV_N = PV(1 + r)^N$
- Future Value (Frequent Compounding in a period):  $FV_N = PV \left(1 + \frac{r_s}{m}\right)^{Nm}$
- Continuous Compounding:  $FV_N = PV e^{r_s N}$
- Future Value of an ordinary annuity:  $FV_N = A \left[ \frac{(1+r)^N - 1}{r} \right]$
- Present Value of an ordinary annuity:  $PV = A \left[ \frac{1 - (1+r)^{-N}}{r} \right]$
- Present value of a annuity due:  $PV = A \{ [1 - (1 + r)^{-N}] / r \} (1 + r)$

7. Present Value of a perpetuity:  $PV = A/r$

8. Growth rate:  $g = \left(\frac{FVN}{PV}\right)^{\frac{1}{N}} - 1$

9.  $NPV = CF_0 + \frac{CF_1}{1+r} + \frac{CF_2}{(1+r)^2} + \dots + \frac{CF_t}{(1+r)^t}$

10. Formulae for confidence Intervals and Hypothesis Testing Concerning Means:

C.I. for Population Mean (Normal Distribution, known Population Variance)	$\bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$
C.I. for Population Mean (Large Sample, unknown Population Variance)	$\bar{X} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$
C.I. for Population Mean (Small Sample, unknown Population Variance)	$\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$
Test Statistic for Population Mean (Population Variance unknown)	$t_{n-1} = \frac{\bar{X} - \mu}{s/\sqrt{n}}$
Test Statistic for Population Mean (Population Variance known)	$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$
Test Statistic for Population Mean (Large Sample, Population Variance unknown)	$z = \frac{\bar{X} - \mu}{s/\sqrt{n}}$
Test Statistic for difference between 2 Population Means (Normal Distribution, Population Variances unknown but assumed equal)	$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$ <p>where <math>s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}</math></p>
Test Statistic for difference between 2 Population Means (Normal Distribution, Population Variances known)	$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
If $n_1 \geq 30$ & $n_2 \geq 30$ , and population variances are unknown, s will replace $\sigma$ .	



$$11. \quad SST = \sum_{i=1}^k \sum_{j=1}^n x_{ij}^2 - \frac{1}{kn} \cdot T^2$$

$$12. \quad SS(Tr) = \frac{1}{n} \cdot \sum_{i=1}^k T_i^2 - \frac{1}{kn} \cdot T^2$$

$$13. \quad SSE = SST - SS(Tr)$$

$$14. \quad r = \frac{n \sum xy - \sum x \sum y}{\sqrt{[n \sum x^2 - (\sum x)^2][n \sum y^2 - (\sum y)^2]}}$$

$$15. \quad \hat{y} = b_0 + b_1 x, \quad b_1 = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}, \quad b_0 = \bar{y} - b_1 \bar{x}$$

Appendix B Table of the Student's *t*-Distribution (One-Tailed Probabilities)

df	<i>p</i> = 0.10	<i>p</i> = 0.05	<i>p</i> = 0.025	<i>p</i> = 0.01	<i>p</i> = 0.005	df	<i>p</i> = 0.10	<i>p</i> = 0.05	<i>p</i> = 0.025	<i>p</i> = 0.01	<i>p</i> = 0.005
1	3.078	6.314	12.706	31.821	63.657	31	1.309	1.696	2.040	2.453	2.744
2	1.886	2.920	4.303	6.965	9.925	32	1.309	1.694	2.037	2.449	2.738
3	1.638	2.353	3.182	4.541	5.841	33	1.308	1.692	2.035	2.445	2.733
4	1.533	2.132	2.776	3.747	4.604	34	1.307	1.691	2.032	2.441	2.728
5	1.476	2.015	2.571	3.365	4.032	35	1.306	1.690	2.030	2.438	2.724
6	1.440	1.943	2.447	3.143	3.707	36	1.306	1.688	2.028	2.434	2.719
7	1.415	1.895	2.365	2.998	3.499	37	1.305	1.687	2.026	2.431	2.715
8	1.397	1.860	2.306	2.896	3.355	38	1.304	1.686	2.024	2.429	2.712
9	1.383	1.833	2.262	2.821	3.250	39	1.304	1.685	2.023	2.426	2.708
10	1.372	1.812	2.228	2.764	3.169	40	1.303	1.684	2.021	2.423	2.704
11	1.363	1.796	2.201	2.718	3.106	41	1.303	1.683	2.020	2.421	2.701
12	1.356	1.782	2.179	2.681	3.055	42	1.302	1.682	2.018	2.418	2.698
13	1.350	1.771	2.160	2.650	3.012	43	1.302	1.681	2.017	2.416	2.695
14	1.345	1.761	2.145	2.624	2.977	44	1.301	1.680	2.015	2.414	2.692
15	1.341	1.753	2.131	2.602	2.947	45	1.301	1.679	2.014	2.412	2.690
16	1.337	1.746	2.120	2.583	2.921	46	1.300	1.679	2.013	2.410	2.687
17	1.333	1.740	2.110	2.567	2.898	47	1.300	1.678	2.012	2.408	2.685
18	1.330	1.734	2.101	2.552	2.878	48	1.299	1.677	2.011	2.407	2.682
19	1.328	1.729	2.093	2.539	2.861	49	1.299	1.677	2.010	2.405	2.680
20	1.325	1.725	2.086	2.528	2.845	50	1.299	1.676	2.009	2.403	2.678
21	1.323	1.721	2.080	2.518	2.831	60	1.296	1.671	2.000	2.390	2.660
22	1.321	1.717	2.074	2.508	2.819	70	1.294	1.667	1.994	2.381	2.648
23	1.319	1.714	2.069	2.500	2.807	80	1.292	1.664	1.990	2.374	2.639
24	1.318	1.711	2.064	2.492	2.797	90	1.291	1.662	1.987	2.368	2.632
25	1.316	1.708	2.060	2.485	2.787	100	1.290	1.660	1.984	2.364	2.626
26	1.315	1.706	2.056	2.479	2.779	110	1.289	1.659	1.982	2.361	2.621
27	1.314	1.703	2.052	2.473	2.771	120	1.289	1.658	1.980	2.358	2.617
28	1.313	1.701	2.048	2.467	2.763	200	1.286	1.653	1.972	2.345	2.601
29	1.311	1.699	2.045	2.462	2.756	∞	1.282	1.645	1.960	2.326	2.576
30	1.310	1.697	2.042	2.457	2.750						

Appendix D Table of the *F*-Distribution  
Panel D. Critical values for right-hand tail area equal to 0.005

		Numerator: <i>df</i> <sub>1</sub> and Denominator: <i>df</i> <sub>2</sub>																							
<i>df</i> <sub>1</sub> :		1	2	3	4	5	6	7	8	9	10	11	12	15	20	21	22	23	24	25	30	40	60	120	∞
<i>df</i> <sub>2</sub> : 1	16211	20000	21615	22500	23056	23437	23715	23925	24091	24222	24334	24426	24630	24836	24863	24892	24915	24940	24959	25044	25146	25253	25359	25464	
2	198.5	199.0	199.2	199.3	199.3	199.4	199.4	199.4	199.4	199.4	199.4	199.4	199.4	199.4	199.4	199.4	199.4	199.4	199.4	199.5	199.5	199.5	199.5	200	
3	55.55	49.80	47.47	46.20	45.39	44.84	44.43	44.13	43.88	43.68	43.52	43.39	43.08	42.78	42.73	42.69	42.66	42.62	42.59	42.47	42.31	42.15	41.99	41.83	
4	31.33	26.28	24.26	23.15	22.46	21.98	21.62	21.35	21.14	20.97	20.82	20.70	20.44	20.17	20.13	20.09	20.06	20.03	20.00	19.89	19.75	19.61	19.47	19.32	
5	22.78	18.31	16.53	15.56	14.94	14.51	14.20	13.96	13.77	13.62	13.49	13.38	13.15	12.90	12.87	12.84	12.81	12.78	12.76	12.66	12.53	12.40	12.27	12.14	
6	18.63	14.54	12.92	12.03	11.46	11.07	10.79	10.57	10.39	10.25	10.13	10.03	9.81	9.59	9.56	9.53	9.50	9.47	9.45	9.36	9.24	9.12	9.00	8.88	
7	16.24	12.40	10.88	10.05	9.52	9.16	8.89	8.68	8.51	8.38	8.27	8.18	7.97	7.75	7.72	7.69	7.67	7.64	7.62	7.53	7.42	7.31	7.19	7.08	
8	14.69	11.04	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.10	7.01	6.81	6.61	6.58	6.55	6.53	6.50	6.48	6.40	6.29	6.18	6.06	5.95	
9	13.61	10.11	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.31	6.23	6.03	5.83	5.80	5.78	5.75	5.73	5.71	5.62	5.52	5.41	5.30	5.19	
10	12.83	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.75	5.66	5.47	5.27	5.25	5.22	5.20	5.17	5.15	5.07	4.97	4.86	4.75	4.64	
11	12.23	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54	5.42	5.32	5.24	5.05	4.86	4.83	4.80	4.78	4.76	4.74	4.65	4.55	4.45	4.34	4.23	
12	11.75	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.99	4.91	4.72	4.53	4.50	4.48	4.45	4.43	4.41	4.33	4.23	4.12	4.01	3.90	
13	11.37	8.19	6.93	6.23	5.79	5.48	5.25	5.08	4.94	4.82	4.72	4.64	4.46	4.27	4.24	4.22	4.19	4.17	4.15	4.07	3.97	3.87	3.76	3.65	
14	11.06	7.92	6.68	6.00	5.56	5.26	5.03	4.86	4.72	4.60	4.51	4.43	4.25	4.06	4.03	4.01	3.98	3.96	3.94	3.86	3.76	3.66	3.55	3.44	
15	10.80	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.33	4.25	4.07	3.88	3.86	3.83	3.81	3.79	3.77	3.69	3.59	3.48	3.37	3.26	
16	10.58	7.51	6.30	5.64	5.21	4.91	4.69	4.52	4.38	4.27	4.18	4.10	3.92	3.73	3.71	3.68	3.66	3.64	3.62	3.54	3.44	3.33	3.22	3.11	
17	10.38	7.35	6.16	5.50	5.07	4.78	4.56	4.39	4.25	4.14	4.05	3.97	3.79	3.61	3.58	3.56	3.53	3.51	3.49	3.41	3.31	3.21	3.10	2.98	
18	10.22	7.21	6.03	5.37	4.96	4.66	4.44	4.28	4.14	4.03	3.94	3.86	3.68	3.50	3.47	3.45	3.42	3.40	3.38	3.30	3.20	3.10	2.99	2.87	
19	10.07	7.09	5.92	5.27	4.85	4.56	4.34	4.18	4.04	3.93	3.84	3.76	3.59	3.40	3.37	3.35	3.33	3.31	3.29	3.21	3.11	3.00	2.89	2.78	
20	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.76	3.68	3.50	3.32	3.29	3.27	3.24	3.22	3.20	3.12	3.02	2.92	2.81	2.69	
21	9.83	6.89	5.73	5.09	4.68	4.39	4.18	4.01	3.88	3.77	3.68	3.60	3.43	3.24	3.22	3.19	3.17	3.15	3.13	3.05	2.95	2.84	2.73	2.61	
22	9.73	6.81	5.65	5.02	4.61	4.32	4.11	3.94	3.81	3.70	3.61	3.54	3.36	3.18	3.15	3.12	3.10	3.08	3.06	2.98	2.88	2.77	2.66	2.55	
23	9.63	6.73	5.58	4.95	4.54	4.26	4.05	3.88	3.75	3.64	3.55	3.47	3.30	3.12	3.09	3.06	3.04	3.02	3.00	2.92	2.82	2.71	2.60	2.48	
24	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	3.59	3.50	3.42	3.25	3.06	3.04	3.01	2.99	2.97	2.95	2.87	2.77	2.66	2.55	2.43	
25	9.48	6.60	5.46	4.84	4.43	4.15	3.94	3.78	3.64	3.54	3.45	3.37	3.20	3.01	2.99	2.96	2.94	2.92	2.90	2.82	2.72	2.61	2.50	2.38	
30	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.25	3.18	3.01	2.82	2.80	2.77	2.75	2.73	2.71	2.63	2.52	2.42	2.30	2.18	
40	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	3.03	2.95	2.78	2.60	2.57	2.55	2.52	2.50	2.48	2.40	2.30	2.18	2.06	1.93	
60	8.49	5.79	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.82	2.74	2.57	2.39	2.36	2.33	2.31	2.29	2.27	2.19	2.08	1.96	1.83	1.69	
120	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.62	2.54	2.37	2.19	2.16	2.13	2.11	2.09	2.07	1.98	1.87	1.75	1.61	1.43	
Infinity	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.43	2.36	2.19	2.00	1.97	1.95	1.92	1.90	1.88	1.79	1.67	1.53	1.36	1.00	



Appendix D Table of the F-Distribution  
Panel C. Critical values for right-hand tail area equal to 0.01

		Numerator: df <sub>j</sub> and Denominator:																							
df <sub>i</sub> :	1	2	3	4	5	6	7	8	9	10	11	12	15	20	21	22	23	24	25	30	40	60	120		
df <sub>i</sub> : 1	4052	5000	5403	5625	5764	5859	5928	5982	6023	6056	6083	6106	6157	6209	6216	6223	6229	6235	6240	6261	6287	6313	6339	63	
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	27.1	26.9	26.7	26.7	26.6	26.6	26.6	26.6	26.5	26.4	26.3	26.2	26.1	26.0
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.5	14.4	14.2	14.0	14.0	14.0	13.9	13.9	13.9	13.8	13.7	13.7	13.6	13.5	13.4
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	10.0	9.89	9.72	9.55	9.53	9.51	9.49	9.47	9.45	9.38	9.29	9.20	9.11	9.01	8.91
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72	7.56	7.40	7.37	7.35	7.33	7.31	7.30	7.23	7.14	7.06	6.97	6.88	6.79
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47	6.31	6.16	6.13	6.11	6.09	6.07	6.06	5.99	5.91	5.82	5.74	5.65	5.56
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67	5.52	5.36	5.34	5.32	5.30	5.28	5.26	5.20	5.12	5.03	4.95	4.86	4.77
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11	4.96	4.81	4.79	4.77	4.75	4.73	4.71	4.65	4.57	4.48	4.40	4.31	4.22
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71	4.56	4.41	4.38	4.36	4.34	4.33	4.31	4.25	4.17	4.08	4.00	3.91	3.82
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40	4.25	4.10	4.08	4.06	4.04	4.02	4.01	3.94	3.86	3.78	3.69	3.60	3.51
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16	4.01	3.86	3.84	3.82	3.80	3.78	3.76	3.70	3.62	3.54	3.45	3.36	3.27
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96	3.82	3.66	3.64	3.62	3.60	3.59	3.57	3.51	3.43	3.34	3.25	3.16	3.07
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.86	3.80	3.66	3.51	3.48	3.46	3.44	3.43	3.41	3.35	3.27	3.18	3.09	3.00	2.91
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67	3.52	3.37	3.35	3.33	3.31	3.29	3.28	3.21	3.13	3.05	2.96	2.87	2.78
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55	3.41	3.26	3.24	3.22	3.20	3.18	3.16	3.10	3.02	2.93	2.84	2.75	2.66
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46	3.31	3.16	3.14	3.12	3.10	3.08	3.07	3.00	2.92	2.83	2.75	2.66	2.57
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37	3.23	3.08	3.05	3.03	3.02	3.00	2.98	2.92	2.84	2.75	2.66	2.57	2.48
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30	3.15	3.00	2.98	2.96	2.94	2.92	2.91	2.84	2.76	2.67	2.58	2.49	2.40
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23	3.09	2.94	2.92	2.90	2.88	2.86	2.84	2.78	2.69	2.61	2.52	2.43	2.34
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.24	3.17	3.03	2.88	2.86	2.84	2.82	2.80	2.79	2.72	2.64	2.55	2.46	2.37	2.28
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12	2.98	2.83	2.81	2.78	2.77	2.75	2.73	2.67	2.58	2.50	2.40	2.31	2.22
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.14	3.07	2.93	2.78	2.76	2.74	2.72	2.70	2.69	2.62	2.54	2.45	2.35	2.26	2.17
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03	2.89	2.74	2.72	2.70	2.68	2.66	2.64	2.58	2.49	2.40	2.31	2.22	2.13
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	3.06	2.99	2.85	2.70	2.68	2.66	2.64	2.62	2.60	2.53	2.45	2.36	2.27	2.18	2.09
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84	2.70	2.55	2.53	2.51	2.49	2.47	2.45	2.39	2.30	2.21	2.11	2.02	1.93
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66	2.52	2.37	2.35	2.33	2.31	2.29	2.27	2.20	2.11	2.02	1.92	1.83	1.74
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50	2.35	2.20	2.17	2.15	2.13	2.12	2.10	2.03	1.94	1.84	1.73	1.64	1.55

Appendix D Table of the F-Distribution  
Panel B. Critical values for right-hand tail area equal to 0.025

		Numerator: df <sub>j</sub> and Denominator:																							
df <sub>i</sub> :	1	2	3	4	5	6	7	8	9	10	11	12	15	20	21	22	23	24	25	30	40	60	120	∞	
df <sub>i</sub> : 1	648	799	864	900	922	937	948	957	963	969	973	977	985	993	994	995	996	997	998	1001	1006	1010	1014	1018	1022
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.41	39.43	39.45	39.45	39.45	39.45	39.46	39.46	39.46	39.47	39.48	39.49	39.50	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.37	14.34	14.25	14.17	14.16	14.14	14.13	14.12	14.12	14.08	14.04	13.99	13.95	13.90	13.85
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.79	8.75	8.66	8.56	8.55	8.53	8.52	8.51	8.50	8.46	8.41	8.36	8.31	8.26	8.21
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.57	6.52	6.43	6.33	6.31	6.30	6.29	6.28	6.27	6.23	6.18	6.12	6.07	6.01	5.95
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.41	5.37	5.27	5.17	5.15	5.14	5.13	5.12	5.11	5.07	5.01	4.96	4.90	4.84	4.78
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.71	4.67	4.57	4.47	4.45	4.44	4.43	4.41	4.40	4.36	4.31	4.25	4.20	4.14	4.08
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.24	4.20	4.10	4.00	3.98	3.97	3.96	3.95	3.94	3.89	3.84	3.78	3.73	3.67	3.61
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.91	3.87	3.77	3.67	3.65	3.64	3.63	3.61	3.60	3.56	3.51	3.45	3.39	3.33	3.27
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.66	3.62	3.52	3.42	3.40	3.39	3.38	3.37	3.35	3.31	3.26	3.20	3.14	3.08	3.02
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.47	3.43	3.33	3.23	3.21	3.20	3.18	3.17	3.16	3.12	3.06	3.00	2.94	2.88	2.82
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.32	3.28	3.18	3.07	3.06	3.04	3.03	3.02	3.01	2.96	2.91	2.85	2.79	2.73	2.67
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.20	3.15	3.05	2.95	2.93	2.92	2.91	2.89	2.88	2.84	2.78	2.72	2.66	2.60	2.54
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.09	3.05	2.95	2.84	2.83	2.81	2.80	2.79	2.78	2.73	2.67	2.61	2.55	2.49	2.43
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	3.01	2.96	2.86	2.76	2.74	2.73	2.71	2.70	2.69	2.64	2.59	2.52	2.46	2.40	2.34
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.93	2.89	2.79	2.68	2.67	2.65	2.64	2.63	2.61	2.57	2.51	2			



Panel A. Critical values for right-hand tail area equal to 0.05

Numerator: df1 and Denominator: df2

df1:	1	2	3	4	5	6	7	8	9	10	11	12	15	20	21	22	23	24	25	30	40	60	120	
df2: 1	161	200	216	225	230	234	237	239	241	242	243	244	246	248	248	249	249	249	249	250	251	252	253	2
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.70	8.66	8.65	8.65	8.64	8.64	8.63	8.62	8.59	8.57	8.55	8.
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.86	5.80	5.79	5.79	5.78	5.77	5.77	5.75	5.72	5.69	5.66	5.
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.62	4.56	4.55	4.54	4.53	4.53	4.52	4.50	4.46	4.43	4.40	4.
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.94	3.87	3.86	3.86	3.85	3.84	3.83	3.81	3.77	3.74	3.70	3.
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57	3.51	3.44	3.43	3.43	3.42	3.41	3.40	3.38	3.34	3.30	3.27	3.
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28	3.22	3.15	3.14	3.13	3.12	3.12	3.11	3.08	3.04	3.01	2.97	2.
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07	3.01	2.94	2.93	2.92	2.91	2.90	2.89	2.86	2.83	2.79	2.75	2.
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.85	2.77	2.76	2.75	2.74	2.73	2.70	2.66	2.62	2.58	2.54	2.
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79	2.72	2.65	2.64	2.63	2.62	2.61	2.60	2.57	2.53	2.49	2.45	2.
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69	2.62	2.54	2.53	2.52	2.51	2.51	2.50	2.47	2.43	2.38	2.34	2.
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60	2.53	2.46	2.45	2.44	2.43	2.42	2.41	2.38	2.34	2.30	2.25	2.
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53	2.46	2.39	2.38	2.37	2.36	2.35	2.34	2.31	2.27	2.22	2.18	2.
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48	2.40	2.33	2.32	2.31	2.30	2.29	2.28	2.25	2.20	2.16	2.11	2.
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42	2.35	2.28	2.26	2.25	2.24	2.24	2.23	2.19	2.15	2.11	2.06	2.
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38	2.31	2.23	2.22	2.21	2.20	2.19	2.18	2.15	2.10	2.06	2.01	1.
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34	2.27	2.19	2.18	2.17	2.16	2.15	2.14	2.11	2.06	2.02	1.97	1.
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31	2.23	2.16	2.14	2.13	2.12	2.11	2.11	2.07	2.03	1.98	1.93	1.
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28	2.20	2.12	2.11	2.10	2.09	2.08	2.07	2.04	1.99	1.95	1.90	1.
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25	2.18	2.10	2.08	2.07	2.06	2.05	2.05	2.01	1.96	1.92	1.87	1.
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23	2.15	2.07	2.06	2.05	2.04	2.03	2.02	1.98	1.94	1.89	1.84	1.
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.24	2.20	2.13	2.05	2.04	2.02	2.01	2.01	2.00	1.96	1.91	1.86	1.81	1.
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.22	2.18	2.11	2.03	2.01	2.00	1.99	1.98	1.97	1.94	1.89	1.84	1.79	1.
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.20	2.16	2.09	2.01	2.00	1.98	1.97	1.96	1.96	1.92	1.87	1.82	1.77	1.
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09	2.01	1.93	1.92	1.91	1.90	1.89	1.88	1.84	1.79	1.74	1.68	1.
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00	1.92	1.84	1.83	1.81	1.80	1.79	1.78	1.74	1.69	1.64	1.58	1.
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92	1.84	1.75	1.73	1.72	1.71	1.70	1.69	1.65	1.59	1.53	1.47	1.
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.87	1.83	1.75	1.66	1.64	1.63	1.62	1.61	1.60	1.55	1.50	1.43	1.35	1.
Infinity	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75	1.67	1.57	1.56	1.54	1.53	1.52	1.51	1.46	1.40	1.33	1.25	1.